

DFTT 49/95
 August 1995
 hep-ph/9509282

NLO QCD corrections to the production of off-shell WW pairs at e^+e^- colliders ¹

Ezio Maina and Marco Pizzio

*Dipartimento di Fisica Teorica, Università di Torino
 and INFN, Sezione di Torino
 v. Giuria 1, 10125 Torino, Italy.*

Abstract

QCD corrections to $e^+e^- \rightarrow WW \rightarrow q_1\bar{q}_1q_2\bar{q}_2$ are computed and presented in a form which allows to impose realistic cuts on the structure of the observed events. QCD radiation substantially modifies the jet-jet invariant mass distributions from which the value of the W mass will be extracted. When the range of allowed jet-jet masses is restricted, as it has been proposed in order to suppress non- WW backgrounds, the total cross section is also deeply affected. If all events are forced to four jets, combining the two partons with smallest invariant mass, and the reconstructed masses are required to satisfy $|M_{Ri} - M_W| \leq 10 \text{ GeV}$ $i=1,2$ the lowest order cross section can be reduced by more than 40%.

¹ Work supported in part by Ministero dell' Università e della Ricerca Scientifica.
 e-mail: maina@to.infn.it, pizzio@to.infn.it

Introduction

One of the main goals of Lep II is the measurement of the W mass with high accuracy, possibly of the order of 50 MeV or less ². Two methods have been singled out as the most promising [2]. The first one is based on the rapid increase of the total cross section at threshold and has been recently studied in detail in ref. [3] where it has been shown that the optimal collider energy is about 161 GeV. The second method relies on the direct reconstruction of the mass from the hadronic decay products of the W using the decay channels

$$W^+W^- \rightarrow q_1\bar{q}_1q_2\bar{q}_2 \quad (1)$$

$$W^+W^- \rightarrow q\bar{q}\ell\nu \quad (2)$$

where $\ell = e, \mu$. The tree-level relevant branching ratios are given in table I.

The threshold method will be the first one to be exploited when Lep II is turned on. In the meantime the number of superconducting cavities will be increased until the center-of-mass energy can be pushed to about 175 GeV where the much larger cross-section will allow M_W to be extracted by direct reconstruction. Preliminary studies indicate that the direct measurement will provide a more precise determination of the W mass than the threshold method if all decay channels are combined, while the two methods are expected to have comparable accuracies if only the semileptonic decays are used for direct reconstruction. The possibility of a further energy upgrade to about 190 GeV is presently under consideration. This higher energy would greatly improve the prospects for the discovery of new particles, in particular of the Higgs boson, without compromising the precision expected for the measurement of M_W .

In the case of semileptonic decays, apart from all the usual difficulties related to measuring jet energies and directions, the main uncertainty in the mass measurement derives from the reconstruction of the unobserved neutrino. These uncertainties can be partially eliminated by using energy-momentum conservation and possibly the approximate equality of the W^+ and W^- masses. In the case of hadronic decays, where in principle all decay products can be detected, there are additional ambiguities which stem from the fact that two decays occur in the same event. Even in the simplest case, in which only four jets are detected, there are three possible ways of pairing the jets. If more jets are present the combinatorics becomes rapidly quite complicated.

It has been proposed to force all events to four jets simply iterating a given reconstruction algorithm until only four clusters are left. It is obvious that such a scheme is not without perils. A jet from, say, the decay of the W^+ can be closer to a jet from the decay of the W^- than to any other jet from the positively charged W . If the stray jet carries large energy, then the reconstructed masses will be quite far from the W mass which is already at present known with an error smaller than 200 MeV [4]. One could imagine that such events could be discarded, at the same time providing a mean of suppressing non- WW background. However, due to the intrinsic width of the W and to all experimental uncertainties, it is impossible to impose very stringent cuts on the difference between the measured jet-jet masses and the true mass without

²For a general introduction to the Standard Model predictions for W -pair production in e^+e^- collisions see [1].

substantially reducing the event rate. A precise estimate of the effect of these mass cuts is necessary in order to assess the final accuracy with which M_W will be measured.

A different proposal has been to simply discard all events with five or more jets. It is clear that this can only be a partial solution at best. It excludes hard, non-collinear gluon emission but it does not suppress soft and/or collinear radiation whose effects need in any case to be studied. Furthermore such a procedure would introduce a dependence of the mass measurement on the invariant mass cut which separates four-jet events from the events with a larger jet multiplicity.

QCD radiation can modify the characteristics of the final state like the distributions of jet momenta, jet-jet invariant masses and opening angles from which the value of the W mass will be extracted. These semi-inclusive quantities, on the other hand, are of calorimetric type, that is infrared and collinear safe. The measured invariant masses, as an example, do not change if a massless parton of momentum p splits into two partons of momentum λp and $(1 - \lambda)p$ or if an additional soft gluon is radiated by any of the hard partons. As a consequence they can be reliably studied in perturbative QCD. It is well known that differential distributions, particularly when the phase space for gluon emission is restricted by experimental cuts, can be more sensitive to higher order corrections than fully inclusive quantities like total cross-sections in which virtual and real contributions cancel to a large degree. It is therefore necessary to include higher order QCD effects into the predictions for WW production and decay in a way which allows to impose realistic cuts on the structure of the observed events. In this paper we present the calculation of the complete $\mathcal{O}(\alpha_f)$ corrections to the three diagrams which describe the basic process $e^+e^- \rightarrow WW \rightarrow q_1\bar{q}_1q_2\bar{q}_2$. We will discuss only fully hadronic decays but the extension to semileptonic decays is trivial.

Recently it has been pointed out [5, 6, 7, 8] that cross-talk between the decay of the two W 's might take place also at energy scales much smaller than those typical of jets through color reconnection phenomena and Bose-Einstein correlations. This could weaken the notion of two separate decays and cast doubts on our ability to reconstruct the masses of the original sources from the decay products, producing potentially large uncertainties. Our understanding of these issues is, at present, extremely poor, and the predictions are strongly model-dependent. In this letter we will leave aside these non-perturbative issues. For a discussion of some aspect of colour reconnection from a perturbative point of view see [9, 10].

Calculation

In n dimensions, $n = 4 - 2\epsilon$, using dimensional regularization, the tree-level amplitude squared for the decay $W \rightarrow q\bar{q}$ is:

$$|M_0(q\bar{q})|^2 = 4Ks(1 - \epsilon) \quad (3)$$

with $K = N_C e^2 e_q^2$ and, labelling the momenta with the particle names, $s = (q + \bar{q})^2$. The corresponding amplitude squared for the decay $W \rightarrow q\bar{q}g$ is:

$$|M(q\bar{q}g)|^2 = 8K'(1 - \epsilon) \left\{ \left(\frac{2s^2}{s_1 s_2} - \frac{2s}{s_1} - \frac{2s}{s_2} + \frac{s_2}{s_1} + \frac{s_1}{s_2} \right) - \epsilon \left(\frac{s_2}{s_1} + \frac{s_1}{s_2} + 2 \right) \right\} \quad (4)$$

with $K' = K C_F g_s^2 \mu^{2\epsilon}$, $s = (q + \bar{q} + g)^2$, $s_1 = (q + g)^2$ and $s_2 = (\bar{q} + g)^2$.

The phase space can be expressed as follows:

$$d\text{Lips}(Q \rightarrow q + \bar{q} + g) = d\text{Lips}(Q \rightarrow q + \bar{q}) \times \frac{s^{3\epsilon-1}}{16\pi^2} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} ds_1 ds_2 \{(s - s_1 - s_2) s s_1 s_2\}^{-\epsilon} \quad (5)$$

after integrating over the azimuthal angle in the center of mass of q and g with \bar{q} on the z -axis.

Integrating over s_1 and s_2 in the region Σ where $\min(s_1, s_2) < \Delta$ one obtains:

$$|M_\Sigma(q\bar{q})|^2 = \frac{s^{3\epsilon-1}}{16\pi^2} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \iint_\Sigma ds_1 ds_2 \{(s - s_1 - s_2) s s_1 s_2\}^{-\epsilon} |M(q\bar{q}g)|^2 \quad (6)$$

$$= |M_0(q\bar{q})|^2 C_F \frac{g_s^2}{4\pi^2} \xi \left\{ \frac{1}{\epsilon^2} + \frac{3}{2\epsilon} - \ln^2 \eta + \ln \left(\frac{\eta}{1-\eta} \right) \left(2\eta - \frac{1}{2}\eta^2 - \frac{3}{2} \right) + \frac{7}{2} + \frac{5}{4}\eta - 2 \text{Li}_2(\eta) - \frac{5}{2} \text{Li}_2(1) \right\} \quad (7)$$

$$= |M_0(q\bar{q})|^2 C_F \frac{g_s^2}{4\pi^2} \xi \left\{ \frac{1}{\epsilon^2} + \frac{3}{2\epsilon} - \ln^2 \eta - \frac{3}{2} \ln \eta + \frac{7}{2} - \frac{5}{2} \text{Li}_2(1) \right\} + \mathcal{O}(\eta \ln \eta) \quad (8)$$

where $\eta = \Delta/s$ and $\xi = \left(\frac{\mu^2}{s}\right)^\epsilon e^{-\epsilon(\gamma - \ln(4\pi))}$. $\text{Li}_2(x)$ is the standard dilogarithm with $\text{Li}_2(1) = \pi^2/6$. Δ separates the region of soft and collinear emission from the region where gluon radiation is considered hard. The dependence of the result on η will be discussed later in detail.

The $\mathcal{O}(\alpha_f)$ one-loop contribution to the W decay is

$$M_V(q\bar{q}) = M_0(q\bar{q}) C_F \frac{g_s^2}{8\pi^2} \xi \left\{ -\frac{1}{\epsilon^2} - \frac{3}{2\epsilon} - 4 + \frac{7}{2} \text{Li}_2(1) - i\pi \left(\frac{1}{\epsilon} + \frac{3}{2} \right) \right\}. \quad (9)$$

Combining the virtual contribution (9) with the integral of the real-emission cross section over the soft and collinear region (6–8) the following simple expression is obtained for the W decay at $\mathcal{O}(\alpha_f)$:

$$|M(W \rightarrow q\bar{q})|^2 = |M_0(q\bar{q})|^2 + 2\text{Re}(M_0^*(q\bar{q})M_V(q\bar{q})) + |M_\Sigma(q\bar{q})|^2 + |M_{\bar{\Sigma}}(q\bar{q})|^2 \quad (10)$$

$$= |M_0(q\bar{q})|^2 (1 + F(\eta)) + |M_{\bar{\Sigma}}(q\bar{q})|^2 \quad (11)$$

$$= |M_0(q\bar{q})|^2 (1 + F'(\eta)) + |M_{\bar{\Sigma}}(q\bar{q})|^2 + \mathcal{O}(\eta \ln \eta) \quad (12)$$

where

$$F(\eta) = \frac{C_F \alpha_s}{\pi} \left\{ -\ln^2 \eta + \ln \left(\frac{\eta}{1-\eta} \right) \left(2\eta - \frac{1}{2}\eta^2 - \frac{3}{2} \right) - \frac{1}{2} + \frac{5}{4}\eta - 2 \text{Li}_2(\eta) + \text{Li}_2(1) \right\}, \quad (13)$$

$$F'(\eta) = \frac{C_F \alpha_s}{\pi} \left(-\ln^2 \eta - \frac{3}{2} \ln \eta - \frac{1}{2} + \text{Li}_2(1) \right). \quad (14)$$

In the previous formulae $|M_{\bar{\Sigma}}(q\bar{q})|^2$ is defined in analogy to (6) as the integral of the real-emission amplitude squared $|M(q\bar{q}g)|^2$ over the hard-gluon region $\bar{\Sigma}$ where

$\min(s_1, s_2) > \Delta$. The expression for the sum of the virtual and soft-collinear contributions has been derived many times before in various contests (see for example [12, 13]) and we are in agreement with previous results, possibly after a simple coupling redefinition.

Exploiting the universality of soft and collinear divergencies³, these results can be directly carried over to $e^+e^- \rightarrow WW \rightarrow q_1\bar{q}_1q_2\bar{q}_2$. At $\mathcal{O}(\alpha_f)$ there is no interference between the decays of the two W 's and the amplitude for $e^+e^- \rightarrow WW \rightarrow q_1\bar{q}_1q_2\bar{q}_2g$ splits into two orthogonal terms M_1 and M_2 describing the emission from the $q_1\bar{q}_1$ pair and the $q_2\bar{q}_2$ pair respectively. The quantity s in (10) in the two terms has to be identified with the invariant mass s_{1g} of the $q_1\bar{q}_1g$ system and the invariant mass s_{2g} of the $q_2\bar{q}_2g$ system respectively. Choosing $\eta_1 = \Delta_1/s_{1g} = \eta_2 = \Delta_2/s_{2g} = \eta$ we can write:

$$|M(WW \rightarrow 4q)|^2 = |M_0|^2 + 2\text{Re}(M_0^*M_V) + |M_\Sigma|^2 + |M_{\bar{\Sigma}}|^2 \quad (15)$$

$$= |M_0|^2(1 + 2F'(\eta)) + |M_{\bar{\Sigma}}|^2 + \mathcal{O}(\eta \ln \eta) \quad (16)$$

In (15) M_0 is the tree-level amplitude and M_V the one-loop $\mathcal{O}(\alpha_f)$ amplitude for $e^+e^- \rightarrow WW \rightarrow 4q$. $|M_\Sigma|^2$ and $|M_{\bar{\Sigma}}|^2$ are obtained integrating the tree-level amplitude squared describing $e^+e^- \rightarrow WW \rightarrow q_1\bar{q}_1q_2\bar{q}_2g$ over the gluon variables as in (6). $|M_\Sigma|^2$ is equal to the sum of the integral of $|M_1|^2$ over the region Σ_1 where $\min((q_1+g)^2, (\bar{q}_1+g)^2) < \Delta_1$ and of the integral of $|M_2|^2$ over the region Σ_2 where $\min((q_2+g)^2, (\bar{q}_2+g)^2) < \Delta_2$. $|M_{\bar{\Sigma}}|^2$ is equal to the integrals of the square of M_1 and M_2 over the complementary regions. These latter integrals and the tree-level matrix element need to be calculated only in four dimensions.

The real emission matrix element squared $|M(X+g)|^2$ factorizes into the product of the corresponding emissionless expression $|M(X)|^2$ times a universal function which describe the radiation of a gluon only in the soft and collinear limit. Therefore in the expression for the sum of the virtual and soft-collinear corrections there are unavoidable inaccuracies of order η , related to the approximation of neglecting the gluon energy or emission angle inside $|M(X)|^2$, and the use of the approximate expression (16) produces no loss in precision. In addition it must be remembered that we are effectively lumping the soft-collinear region in the phase space point corresponding to the absence of gluon radiation. Therefore, in order to obtain an accurate determination of the total cross section and in order to avoid disrupting the shape of the distributions of the different observable quantities a small value of η must be chosen. In all our figures we have used $\eta = 1. \times 10^{-4}$ and we have checked that our result depend extremely weakly on the value of η in the range $5. \times 10^{-5} \leq \eta \leq 1. \times 10^{-3}$.

Taking advantage of the fact that close to the appropriate soft-collinear regions, which contribute most to the result, the singular part of the integrands behaves like $|M(q\bar{q}g)|^2$ (4) we can use $y_{1,i} = \ln(q_i + g)^2$ and $y_{2,i} = \ln(\bar{q}_i + g)^2$ as integration variables. In this way it becomes possible to evaluate the real emission cross section using a reasonable amount of CPU time, producing theoretical predictions of an accuracy adequate to the expected precision of the measurement of the W mass.

In the case of real emission, in order to extract two candidate masses from the event,

³The formalism is discussed in detail in ref. [11].

the two partons with smallest [14]

$$y = \frac{2E_i E_j (1 - \cos \theta_{ij})}{s} \quad (17)$$

are merged, summing the corresponding four-momenta. This corresponds, at the present order in perturbation theory, to forcing all events to four clusters before attempting to reconstruct the masses of the two W bosons. In this paper we have restricted our attention to the simple JADE scheme and it remains to be studied whether a different reconstruction scheme [15, 16] might prove more appropriate. Finally, the reconstructed masses are determined, out of the three possible combinations, choosing the two two-jet pairs whose masses M_{R1} and M_{R2} minimize

$$\Delta_M = |M_{R1} - M_W| + |M_{R2} - M_W|. \quad (18)$$

It is obvious that the precise form of the metric which defines the distance in jet-space is to some extent arbitrary. For instance one could use

$$\Delta'_M = (M_{R1} - M_W)^2 + (M_{R2} - M_W)^2. \quad (19)$$

We have checked that at $\sqrt{s} = 160$ GeV and at $\sqrt{s} = 175$ GeV the cross sections obtained with (18) and those obtained with (19) differ by less than 1%.

The expressions presented in this section can be applied without modification to all four-quark neutral current processes $e^+e^- \rightarrow ZZ, Z\gamma, \gamma\gamma \rightarrow q_1\bar{q}_1q_2\bar{q}_2$.

In order to expose the effect of $\mathcal{O}(\alpha_s)$ corrections we have made a number of simplifying assumptions. (1) Initial-state-radiation (ISR) effects have been neglected. It would however be straightforward to include them using standard techniques [17]. (2) Coulomb corrections have not been included although, since they can be expressed as a multiplicative factor times the lowest order (LO) matrix element squared, their inclusion would be rather simple [18]. (3) Quark masses have been neglected since the contribution of b -quarks is severely suppressed by the smallness of the V_{bc} element of the CKM matrix and the c -quark mass is so small compared with M_W . (4) We have not taken into account non-resonant electroweak contributions and QCD four-jet backgrounds [19, 20, 3]. The former has been shown to be well below the per-cent level⁴ (with the possible exception of final states including electrons or electron-neutrinos). The latter, which is dominated by $q\bar{q}gg$ production, is strongly suppressed when two jet pairs with masses close to M_W are required. In ref. [3] the QCD background, with $y_{cut} = 0.01$ and requiring two jet-jet masses within 10 GeV of M_W , has been estimated at about 0.1 pb for $\sqrt{s} = 161$ GeV. Since the QCD background scales only as s^{-1} this estimate can be considered valid, as a first approximation, in the full range of operation of Lep II. However this procedure for reducing the four-jet background has far reaching consequences on the WW signal as will be discussed at length in the following. (5) Non-perturbative colour-reconnection and Bose-Einstein effects have not been considered. (6) We have ignored all experimental uncertainties in the reconstruction of jet momenta and directions which will have to be studied with a full detector simulation.

⁴However they are intimately related to the gauge invariance of the theory. For a detailed discussion of this issue see [1, 21, 22].

All tree-level matrix elements have been computed, in the unitary gauge, using the formalism presented in ref. [23] with the help of a set of routines, called PHACT [24], which generate the building blocks of the helicity amplitudes semi-automatically. The matrix element for $e^+e^- \rightarrow WW \rightarrow q_1\bar{q}_1q_2\bar{q}_2g$ has been first computed in ref. [26]. Our result has been checked against the corresponding amplitude generated by Madgraph [25].

The numerical values of the Standard Model parameters used in the calculation are given in Table II.

Results

Our results are presented in fig.1, fig.2, Table III and Table IV. The total hadronic cross sections at tree-level and at next-to-leading-order (NLO) in QCD are given in the first column of Table III for $\sqrt{s} = 160$ GeV and 175 GeV. These results are sensitive to events with jet pairs of arbitrarily large and small invariant masses within the kinematical limits. In practice events with jet-jet masses too far from the W mass would not be accepted as WW events. Cuts of this sort have been suggested in order to eliminate most of the four-jet QCD background. We therefore require that each of the reconstructed masses, selected according to (18), satisfies

$$|M_{Ri} - M_W| \leq \delta, \quad i = 1, 2. \quad (20)$$

In the second and third column of Table III we show the cross section obtained with $\delta = 30$ GeV and $\delta = 10$ GeV respectively. In parentheses we give the cross sections obtained when all events which are identified as containing five jets, with $y_{cut} = 1. \times 10^{-2}$ in the JADE scheme, are discarded. The ratio of the corresponding five-jet cross section to the total hadronic cross section

$$R_5 = \frac{\sigma(WW \rightarrow 5j)}{\sigma(WW \rightarrow had)} \quad (21)$$

is about 22% at $\sqrt{s} = 175$ GeV with no mass cut.

Let us mention that in the so called ADLO-TH set of cuts, the semi-realistic set which the Working Groups on LEP II Physics have agreed on in order to provide a common ground for the comparison between theoretical calculations and the simulations performed by the experiments, two jets are considered as distinguishable if the invariant mass of the pair is larger than 5 GeV. This corresponds to $y_{cut} \approx 1. \times 10^{-3}$. However at this value of y_{cut} the five-jet cross section is larger than the total hadronic cross section and, as a consequence, the cross section for events with at most four jets is negative. This can be readily seen from eq. (14). If we neglect for simplicity the interplay between the two decays, we have that $y_{cut} = 1. \times 10^{-3}$, since $\sqrt{s} \approx 2M_W$, corresponds to $\eta \approx 4. \times 10^{-3}$ and one finds that, with $\alpha_s = .117$, $F'(0.004) < -1$.

In the narrow width, or stable W 's, approximation the QCD correction to the total cross section is simply twice the correction to the W decay, namely the Born cross section gets multiplied by $(1 + 2\alpha_s/\pi)$. In principle, for off-mass-shell W 's, one could expect additional terms of order $(\alpha_s\Gamma_W)/(\pi M_W) \approx 1. \times 10^{-3}$. In order to study effects of this order of magnitude, all matrix elements have been integrated using VEGAS with

a relative precision not worse than $1. \times 10^{-4}$. We have found that the corrections to the total hadronic cross section for unstable W 's do not differ from those for stable W 's by more than one part in a thousand. Therefore the narrow width approximation appears to be perfectly adequate for the QCD correction to the total cross section⁵. This result however relies on the delicate cancellation between real and virtual contributions, and is strongly modified by any constraint which decreases the phase space available for real emission. This is evident from the second and third column of Table III. At $\sqrt{s} = 175$ GeV, with $\delta = 10$ GeV, NLO QCD corrections decrease the corresponding tree-level cross section by about 33%, which is to be contrasted with the increase of about 7% they produce in the total hadronic cross section in column one. Even with the much milder cut $\delta = 30$ GeV the LO result is decreased by about 11%. At $\sqrt{s} = 160$ GeV, about 200 MeV above the nominal threshold with the value of M_W adopted in this paper, the NLO result with $\delta = 10$ (30) GeV is 45% (15%) smaller than the LO prediction without mass cuts. The corresponding cross section is decreased by approximately 0.8 (0.3) *pb*. It should be remembered that a change in the total cross section of 0.2 *pb* corresponds to a shift of about 100 MeV in the measured W mass [3]. Therefore QCD corrections, in contradiction to the naive expectations based on the behaviour of the total cross section, are potentially as large as the corrections due to ISR and much larger than those produced by the Coulomb interaction, even though they only affect about half of the total $e^+e^- \rightarrow W^+W^-$ cross section. Obviously, any modification of the LO cross section which can be reliably computed can be incorporated into the theoretical predictions which are fitted to the data and do not degrade the expected accuracy of the measurement of M_W . They however directly affect the statistical error and the uncertainty in the theoretical analysis has to be included in the evaluation of the systematic error.

The behaviour of the LO and NLO hadronic cross section as a function of the center-of-mass energy can be found in fig.1. As in Table III we present the total hadronic cross sections at tree-level (dashed curve) and at NLO (continuous curve). The long-dash-short-dash curve and the dash-dot curve show the NLO cross sections obtained with $\delta = 30$ GeV and $\delta = 10$ GeV respectively. The dotted line gives the tree-level cross section with $\delta = 10$ GeV. The analogous cross section with $\delta = 30$ GeV, as expected from the fact that in this case $\delta \gg \Gamma_W$, differs very little from the total cross section and we omit it.

The effect of the mass cut (20) reaches a maximum at about $\sqrt{s} = 155$ GeV and decreases with increasing center-of-mass energy. We notice that at $\sqrt{s} = 150$ GeV, with $\delta = 10$ GeV, the NLO result becomes larger than the LO one. This is due to the fact that typically, below threshold, one of the W 's is on mass-shell while the other is highly virtual. Therefore at tree-level most of the events fail the cut. In this case the rearrangement of invariant masses introduced by QCD radiation blurs the separation between on-mass-shell and off-mass-shell W 's and reduces the effectiveness of the mass cut. From fig.1 we read that the NLO order result with $\delta = 30$ GeV is always considerably smaller the tree-level prediction, confirming that even moderate restrictions on the structure of the final state can produce sizable effects. Choosing $\delta = 10$ GeV decreases the available event rate by a factor close to 50% and might

⁵As a consequence, the QCD corrected cross section for $W^+W^- \rightarrow q\bar{q}\ell\nu$, if no restriction is imposed on the hadronic part of the final state, is $\sigma_0(1 + \alpha_s/\pi)$ where σ_0 is the Born result.

result in a substantial increase in the statistical error of the W -mass measurement. Just above threshold the QCD corrections depend only weakly on the center-of-mass energy. At $\sqrt{s} = 160$ GeV they amount to 45% (15%) with $\delta = 10$ (30) GeV while at $\sqrt{s} = 165$ they become 38% (14%). Therefore we do not expect the determination of the optimal energy for the threshold measurement of M_W of ref. [3] to be modified by QCD effects.

In fig.2 we present the distribution of the average of the two reconstructed W masses

$$\overline{M} = \frac{1}{2} (M_{R1} + M_{R2}). \quad (22)$$

which has been extensively used as an estimator of M_W . At tree-level the difference between \overline{M} and the average of the masses of the two virtual W 's, namely the masses which one would reconstruct if the quarks could always be correctly paired, is extremely small. Clearly one could consider more sophisticated approaches, for instance using the mass distribution in the (M_{R1}, M_{R2}) plane but this analysis is beyond the scope of this letter. The two upper curves in fig.2a are obtained when all events are retained, even when one or both the candidate masses are far from M_W . The results for $\delta = 30$ GeV and $\delta = 10$ GeV are shown in fig.2b and fig.2c respectively. The dashed lines are the LO result while the NLO predictions are given by the continuous lines. The NLO cross section for events with at most four jets, at $y_{cut} = 1. \times 10^{-2}$ is shown by the dotted lines. When no cut on $|M_{Ri} - M_W|$ is imposed, at NLO the peak cross section is strongly reduced with respect to the LO distribution and a large tail at low masses is generated. As δ decreases the tail gradually disappears while the number of events contained in the peak at about the W mass decreases only slightly. It is precisely the distortion of the average-mass tree-level spectrum that explains the dramatic effect of the mass cut (20) at NLO. We see that the low-mass tail persists also in the lower multiplicity sample and therefore it is not predominantly populated by five-jet events.

In an attempt to quantify the modifications of the average mass distribution which are induced by QCD corrections we present in Table IV the average deviation, $\langle \overline{M} - M_W \rangle$ and the root-mean-square (RMS) deviation, $\langle (\overline{M} - M_W)^2 \rangle^{\frac{1}{2}}$, of the mass distribution with respect to M_W in GeV. In each column the left (right) hand side value refers to the LO (NLO). In the first row all events are accepted while in the second and third row the two jet-jet masses satisfy $|M_{Ri} - M_W| \leq \delta, i = 1, 2$ with $\delta = 30$ GeV and $\delta = 10$ GeV respectively. The results in the last three rows are obtained discarding all events which are recognized as five jets with $y_{cut} = 1. \times 10^{-2}$ in the JADE scheme.

Table IV shows that already at tree-level the average reconstructed mass \overline{M} is smaller than M_W . The difference $\Delta M = M_W - \overline{M}$ depends on the mass cut. ΔM can be larger than 300 MeV and decreases with δ down to about 40 MeV at $\delta = 10$ GeV. This behaviour can be observed directly in fig.2. The asymmetry which is evident in fig.2a decreases as the mass cut is tightened. When no mass cut is imposed the mass distribution is so deeply altered that it is almost meaningless to compare the tree-level two lowest order moments with those at NLO. This is still partially true for $\delta = 30$ GeV. The low mass tail is non-negligible and considerably increase the mass shift from .274 MeV to 1.70 GeV and the RMS width of the distribution, from 2.39 GeV to 5.11 GeV. When the mass cut is $\delta = 10$ GeV ΔM increases from 41 MeV to 125 MeV while the RMS deviation increase from about 1.7 GeV to slightly more than 2 GeV.

We believe that the impact of the results presented in this paper on the precision measurement of M_W , both at threshold and through direct reconstruction, should be carefully assessed taking into account the other known sources of large corrections, namely ISR and Coulomb interaction, and the experimental uncertainties. In our opinion an improvement of our understanding of the four-jet QCD background and a refinement of the strategies to reduce it are required. As for many other effects, if the theoretical analysis can be made sufficiently precise and, at the same time, accurate data are available, through a careful extrapolation of LEP I results, or can be obtained, possibly through a dedicated run below threshold, QCD backgrounds could become part of the predictions without compromising the accuracy with which the W -mass will be measured. Further work will be necessary to determine which combination of cuts will result in the highest accuracy on M_W , balancing the smaller number of events with a decreased level of background.

Conclusions

We have presented the calculation of the complete $\mathcal{O}(\alpha_f)$ corrections to $e^+e^- \rightarrow WW \rightarrow q_1\bar{q}_1q_2\bar{q}_2$ in a form which allows to impose realistic cuts on the structure of the observed events. It has been shown that QCD radiation modifies in an important way the distribution of jet-jet invariant masses and therefore that it strongly influences the measurement of the W mass from direct reconstruction of the decay products. For total hadronic cross-sections virtual and real contributions cancel to a large degree if all events are retained. If however only events with two jet-jet masses close to M_W are accepted, higher order QCD corrections can substantially decrease the cross section.

A number of issues have not been considered in this paper and will have to be addressed in order to obtain a more complete theoretical understanding of the production of WW pairs at Lep II and to determine the best strategies for the precision measurement of the W mass. (1) The precise choice of the distribution or combination of variables from which to extract M_W . (2) The influence of the jet reconstruction algorithm including hadronization. (3) Initial state radiation. (4) Coulomb corrections. (5) Non resonant background effects. (6) Colour reconnection effects. As already mentioned, initial state radiation and Coulomb corrections are relatively straightforward and an improved analysis which include both effects is well under way.

References

- [1] W. Beenakker and A. Denner, Int. J. Mod. Phys. **A9** (1994) 4837.
- [2] *Measurement of the W mass at LEP 200: report from the working group on mass measurement*, J. Bijnenes et al., Aachen ECFA Workshop (1986);
Report from the working group on LEP 200 physics, S. Katsanevas et al., DELPHI 92–166 PHYS250 (1992).
- [3] W.J. Stirling, University of Durham preprint DTP/95/24 (1995).
- [4] *Measurement of the W boson mass from CDF*, talk presented by Young–Kee Kim at the 10th Topical Workshop on Proton–Antiproton Collider Physics, Batavia, IL, 9–13 May 1995, FERMILAB–CONF–95–240–E.
- [5] G. Gustafson, U. Petterson and P. Zerwas, Phys. Lett. **B209** (1988) 90.
- [6] T. Sjöstrand and V.A. Khoze, Z. Phys. **C 62** (1994) 281.
- [7] G. Gustafson and J. Häkkinen, Z. Phys. **C 64** (1994) 659.
- [8] L. Lonnblad and T. Sjöstrand, Phys. Lett. **B351** (1995) 293.
- [9] E. Accomando, A. Ballestrero and E. Maina, U. of Torino preprint DFTT 37/95, to be published in Phys. Lett B.
- [10] A. Ballestrero, V.A. Khoze, E. Maina, S. Moretti and W.J. Stirling, DFTT 04/95 and DTP/95/10, August 1995.
- [11] W.T. Giele and E.W.N. Glover, Phys. Rev. **D46** (1992) 1980.
- [12] E.W.N. Glover and W.J. Stirling, Phys. Lett. **B295** (1992) 128.
- [13] G. Kramer and B. Lampe, Fortschr. Phys. **37** (1989) 161.
- [14] JADE Collaboration, W. Bartel *et al.*, Z. Phys. **C 33** (1986) 23;
JADE Collaboration, S. Bethke *et al.*, Phys. Lett. **B213** (1988) 235.
- [15] N. Brown and W.J. Stirling, Phys. Lett. **B252** (1990) 657;
N. Brown and W.J. Stirling, Z. Phys. **C 53** (1992) 629.
- [16] S. Bethke, Z. Kunszt, D.E. Soper and W.J. Stirling, Nucl. Phys. **B370** (1992) 310.
- [17] E.A. Kuraev and V.S. Fadin, Yad. Fiz. **41** (1985) 733;
F.A. Berends, G. Burgers and W.L. van Neerven, Nucl. Phys. **B297** (1988) 429;
B304 (1988) 921(E);
M. Cacciari, A. Deandrea, G. Montagna and O. Nicrosini, Z. Phys. **C 52** (1991) 421;
D. Bardin, M. Bilenkii, A. Olchevskii and T. Riemann, Phys. Lett. **B308** (1994) 344;
G. Montagna, O. Nicrosini, G. Passarino and F. Piccinini, Phys. Lett. **B348** (1995) 178.

- [18] V.S. Fadin, V.A. Khoze and A.D. Martin, Phys. Lett. **B311** (1993) 311;
D. Bardin, W. Beenakker and A. Denner, Phys. Lett. **B317** (1993) 213;
V.S. Fadin, V.A. Khoze, A.D. Martin and A. Chapovsky, Phys. Rev. **D52** (1995) 1377;
V.A. Khoze and W.J. Stirling, Phys. Lett. **B356** (1995) 373;
V.S. Fadin, V.A. Khoze, A.D. Martin and W.J. Stirling, University of Durham preprint DTP/95/64 (1995).
- [19] D. Bardin, M. Bilenkii, D. Lehner, A. Olchevskii and T. Riemann, Nucl. Phys. B, Proc. Suppl. **37B** (1994) 148.
- [20] F.A. Berends, R. Pittau and R. Kleiss, Nucl. Phys. **B424** (1994) 308; Nucl. Phys. **B426** (1994) 344;
R. Pittau, Phys. Lett. **B335** (1994) 490.
- [21] A. Aeppli, F. Cuyppers, G.J. van Oldenborgh, Phys. Lett. **B314** (1993) 413.
- [22] E.N. Argyres, W. Beenakker, G.J. van Oldenborgh, A. Denner, S. Dittmaier, J. Hoogland, R. Kleiss, C.P. Papadopoulos and G. Passarino, University of Leiden preprint, INLO-PUB-8/95.
- [23] A. Ballestrero and E. Maina, Phys. Lett. **B350** (1995) 225.
- [24] A. Ballestrero, in preparation.
- [25] T. Stelzer and W.F. Long, Comp.Phys. Commun. **81** (1994) 357.
- [26] N. Brown, Z. Phys. **C 51** (1991) 107.

Figure Captions

Fig.1 Hadronic cross section for $e^+e^- \rightarrow W^+W^-$ as a function of \sqrt{s} . The dashed line and the dotted line are tree-level predictions. For the latter we have required $|M_{Ri} - M_W| \leq 10$ GeV, $i = 1, 2$. The continuous line is the total hadronic NLO result. The long-dash-short-dash curve and the dash-dot curve show the NLO cross sections obtained with $\delta = 30$ GeV and $\delta = 10$ GeV respectively. ISR and Coulomb effects are not included. The parameter values are given in Table II.

Fig.2 Distribution of the average of the two reconstructed mass $\overline{M} = \frac{1}{2}(M_{R1} + M_{R2})$. The dashed lines are the tree-level predictions. The continuous lines and the dotted lines include NLO QCD corrections. For the latter five-jet events, defined using $y_{cut} = 1. \times 10^{-2}$ in the JADE scheme, have been excluded. In fig.2a no cut on $|M_{Ri} - M_W|$ has been imposed. In fig.2b and fig.2c we have required $|M_{Ri} - M_W| \leq 10$ GeV $i = 1, 2$ with $\delta = 30$ GeV and $\delta = 10$ GeV respectively. ISR and Coulomb effects are not included. The parameter values are given in Table II.

Table Captions

Table I Branching ratios of the WW pairs at tree-level. The decay channels involving τ 's are particularly challenging and are usually analyzed separately.

Table II Parameters used in the numerical part of the paper.

Table III Total hadronic cross sections. For each energy the upper line refers to the LO and the lower one to the results at NLO in QCD. In the first column all events are accepted while in the second and third the two jet-jet masses satisfy $|M_{Ri} - M_W| \leq \delta, i = 1, 2$ with $\delta = 30$ GeV and $\delta = 10$ GeV respectively. The numbers in parentheses are obtained discarding all events which are recognized as five jets with $y_{cut} = 1. \times 10^{-2}$ in the JADE scheme.

Table IV Average deviation and RMS deviation from M_W in GeV. In each column the left hand side value refers to the LO and the right hand side one to the results at NLO in QCD. In the first row all events are accepted while in the second and third row the two jet-jet masses satisfy $|M_{Ri} - M_W| \leq \delta, i = 1, 2$ with $\delta = 30$ GeV and $\delta = 10$ GeV respectively. The results in the last three rows are obtained discarding all events which are recognized as five jets with $y_{cut} = 1. \times 10^{-2}$ in the JADE scheme.

$W^+W^- \rightarrow q_1\bar{q}_1q_2\bar{q}_2$	$\frac{4}{9}$
$W^+W^- \rightarrow q\bar{q}\ell\nu$	$\frac{8}{27}$
$W^+W^- \rightarrow \ell\nu\ell'\nu'$	$\frac{4}{81}$
$W^+W^- \rightarrow \tau + \text{anything}$	$\frac{17}{81}$

Table I

parameter	value
M_Z	91.173 GeV
$M_W = M_Z \cos \theta_W$	79.905 GeV
Γ_Z	2.49 GeV
Γ_W	2.08 GeV
α^{-1}	128.
$\sin^2 \theta_W$	0.2319
α_s	.117
$(\hbar c)^2$.38937966 10^9 pbGeV ²

Table II

		$\delta = 30$ GeV	$\delta = 10$ GeV
$\sqrt{s} = 175$ GeV ($y_{cut} = 10^{-2}$)	6.749	6.725	6.393
	7.251 (5.638)	6.169 (4.718)	4.277 (3.383)
$\sqrt{s} = 160$ GeV	1.784	1.757	1.513
	1.916	1.513	0.980

Table III

$\sqrt{s} = 175 \text{ GeV}$	$\langle \overline{M} - M_W \rangle$		$\langle (\overline{M} - M_W)^2 \rangle^{\frac{1}{2}}$	
	LO	NLO	LO	NLO
no cut	-0.340	-4.82	2.645	10.65
$\delta = 30 \text{ GeV}$	-0.274	-1.70	2.390	5.11
$\delta = 10 \text{ GeV}$	-0.041	-0.125	1.679	2.02
$y_{cut} = 0.01$	-0.340	-3.88	2.645	9.88
$\delta = 30 \text{ GeV}, y_{cut} = 0.01$	-0.274	-1.09	2.390	4.15
$\delta = 10 \text{ GeV}, y_{cut} = 0.01$	-0.041	-0.092	1.679	1.78

Table IV

Fig.1

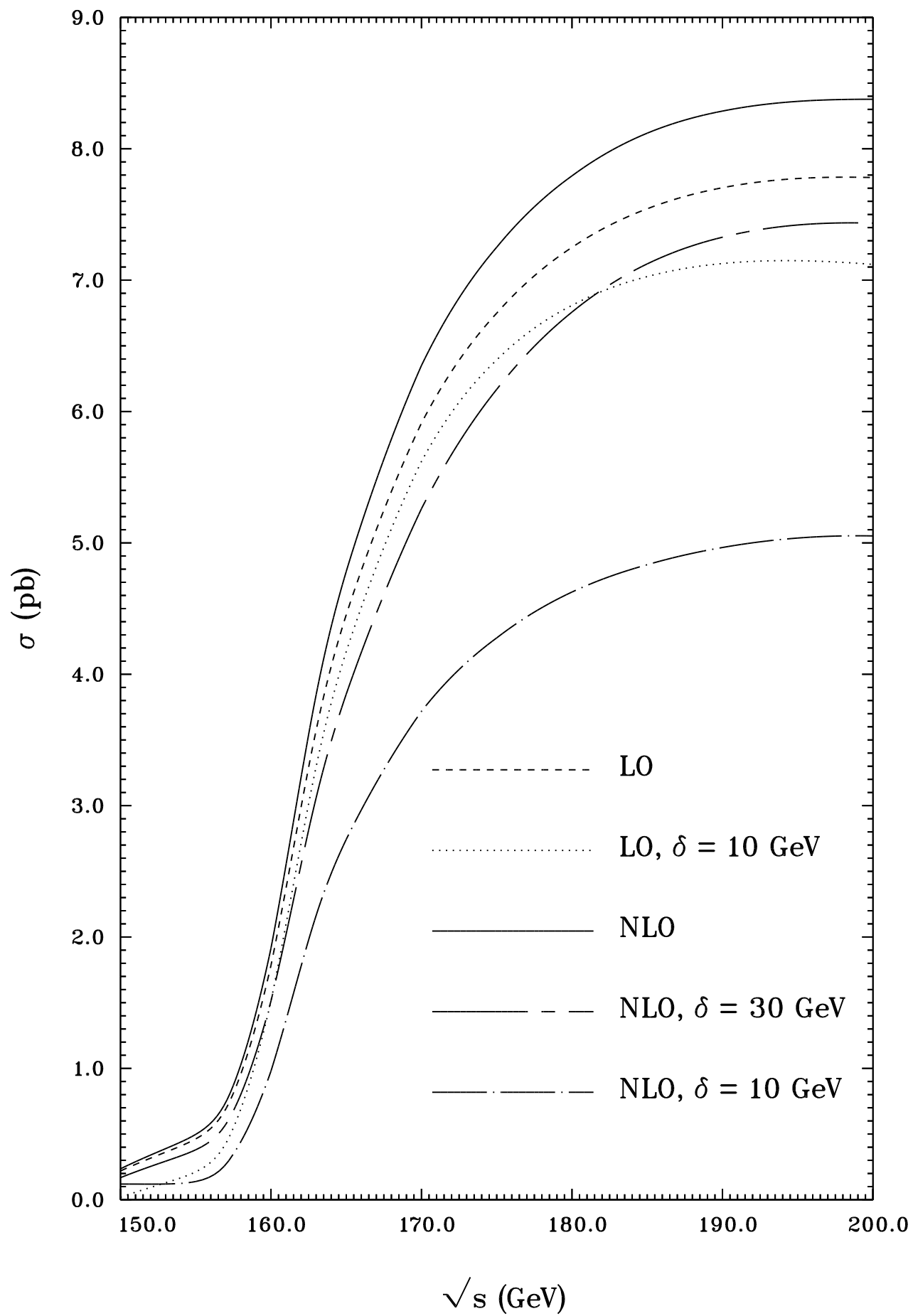


Fig.2

